The Cost of Continuations
Internship Proposal

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**Background.** The mathematical model behind functional programming languages proof assistants is the λ-calculus. It is an elegant machine-independent formalism, deeply connected to the proof theory of Intuitionistic Logic and having just one computational rule, β-reduction. Its distance from low-level details is both its advantage and its drawback: it allows to write concise and modular programs but it is not clear how to measure the complexity of λ-terms. The problem is that β-reduction does not look as an atomic computational rule.

Can we do complexity theory in the λ-calculus? Such a fundamental question has been neglected for a long time. Essentially, to answer positively one needs to prove that the number of β-steps is a reasonable time cost models, that is, it is a measure polynomially related to the number of transitions of Turing machines. For the λ-calculus the existence of an evaluation strategy whose number of steps is a reasonable cost model has been a long-standing open problem. Positive answers to special cases were known since 1995 [BG95], but a solution for the general case has been provided only in 2014, by Accattoli and Dal Lago [AL14]. The techniques developed for that result, inspired by linear logic, have then been employed to show that the most common strategies in the λ-calculus (call-by-name, call-by-value, call-by-need) are all reasonable, that is, their number of steps can be taken as a reasonable cost model for time [ABM14, AC15, ABM15].

**The topic of the Internship.** The aim of the internship is to extend these results to λ-calculi with first-class continuations. Since the seminal work of Griffin [Grif90], it is known that these settings arise as programming languages typed by the formulae of Classical Logic, and they all extend the λ-calculus with constructors for continuations and rewriting rules for their propagation. The intuition is that the cost of such additional operations is negligible with respect to the cost of β-reduction, because β-dominates. Precisely, the conjecture is that their cost is polynomial in the number of β-steps.

First-class continuations come in many flavors. They can be delimited or undelimited. They may come with an operational semantics (that is, rewriting rules) or with a continuation-passing-style (CPS) semantics (i.e., they are compiled away by translation in the traditional λ-calculus). And even when one fixes a specific flavor, there usually are many different formalisms to present it. The plan of the internship is to show that, no matter the flavor, the cost of continuations is always negligible with respect to the cost of β-reduction. To use a slogan, continuations are for free. Concretely, we will fix a few representative examples and try to prove the conjecture we are interested in.

**Details About the Internship.** The internship will take place in the INRIA Parsifal team, and is meant to be an introduction to the research in many different and yet related topics, such as
continuations, abstract machines, rewriting, complexity analysis, linear logic, etc.

The topic is part of a new research project, COCA HOLA [Acc16], and, if successful may lead to a PhD scholarship funded by the project.

The candidate is supposed to know the basics of $\lambda$-calculus and to have close interactions (almost on a daily basis) with the supervisor.

References


